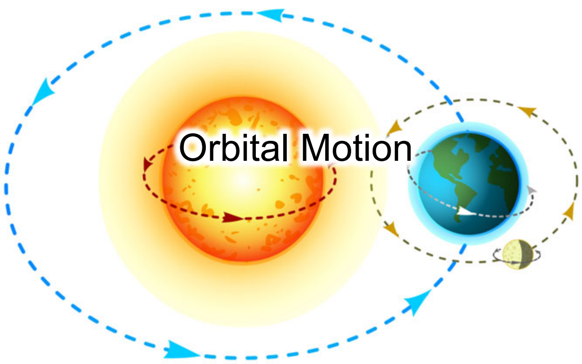
Orbital Motion



LuckySoul (Adobe Stock)

- The orbital motion of all celestial bodies in the universe are governed by gravitational force.
- Many orbits can be approximated as a class of orbit having the following characteristics:
  - A small mass m orbits a much larger mass M.
  - The system is isolated from other masses.

Kepler’s Laws of Planetary Motion

- Tycho Brahe (Danish)
  - Made accurate and comprehensive astronomical observations

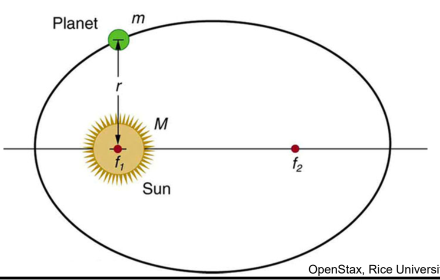
Tycho Brahe (public domain)

- Johannes Kepler (German)
  - Worked with Brahe and devised laws that describe the motion of planets after careful study (over some 20 years) of Brahe's data

Johannes Kepler (public domain)

Kepler's First Law

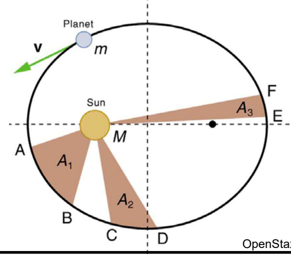
- The orbit of each planet about the Sun is an ellipse with the Sun at one focus.



OpenStax, Rice University (CC BY 4.0)

Kepler's Second Law

- Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times.



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Kepler's Third Law

- The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun.

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

- Kepler's laws, based on observations, explained what was happening.
- Newton showed, through his law of universal gravitation, that gravitational force was the cause.
  - Newton provided a theoretical basis for Kepler's third law.

Deriving Kepler's Third Law

- Consider a small mass m in a circular orbit about a large mass M.

$$F_{net} = ma_c = \frac{mv^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_{net} = \frac{m4\pi^2 r^2}{T^2 r} = \frac{m4\pi^2 r}{T^2}$$

The net external force is caused by gravity.

$$G\frac{mM}{r^2} = \frac{m4\pi^2 r}{T^2}$$

Solving for T^2 gives

$$T^2 = \frac{4\pi^2}{GM} r^3$$

For 2 different satellites

$$T_1^2 = \frac{4\pi^2}{GM} r_1^3 \qquad T_2^2 = \frac{4\pi^2}{GM} r_2^3$$

Taking the ratio between the two satellites gives

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

This is Kepler's third law.

Note that Kepler's third law is valid only for comparing satellites of the same parent body, because only then does the mass of the parent body M cancel.

If we solve $T^2 = \frac{4\pi^2}{GM} r^3$ for the ratio $\frac{T^2}{r^3}$ we get

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

The ratio $\frac{T^2}{r^3}$ is a constant for objects orbiting the same body.